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Eco-Friendly Demand, Carbon Cap-and-Trade and Preservation Technology in a Partially Backlogged Inventory System with Pythagorean Fuzzy Costs: Profit Structure and Parameter Leverage

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Abstract

Firms that introduce products that are environmentally friendly are subjected to a demand that is not only influenced by price and advertising but also by product contents. In this paper, an inventory model is analysed where demand is influenced by the level of stock, the price of selling the product, the number of advertisements and the mixture of herbal and chemical ingredients, deterioration is reduced due to preservation spending, shortages are partly backlogged, the carbon emissions of set-up, holding and deterioration are taxed over a cap, the cost parameters are triangular Pythagorean fuzzy numbers that are defuzzified by a probability-density construction, and the green investment lowers carbon emissions. We calculate the structure of the profit – how much of the 12.9 per cent profit uplift associated with the sustainability measures is due to preservation and how much to green investment, how fast green investment gets saturated and which parameters have leverage. We calculate that about 91 per cent of the uplift can be explained by the preservation and related effects, with only about 9 per cent coming from the green investment, whose marginal return decreases with the unit of investment from US$18.7 up to a point of optimality, after which it turns negative. Profit is extremely sensitive to selling price (+4.1), purchase cost (−3.2), price sensitivity (−3.1) but it is moderately elastic in response to advertising impact (+2.0) and demand scale (+1.2) and almost inelastic in response to carbon tax, emission factors and deterioration cost. The attitude parameter of the fuzzy defuzzification is seen to shift the profit by a linear amount of approximately US$57 per 0.2 increase in the value of the attitude parameter and a higher ratio of herbs to chemicals increases the profit nearly by a factor of three over the range studied. Analysis shows where a company with a new product that is designed to be eco-friendly ought to focus its efforts, and also uncovers a counterintuitive result from the carbon tax that needs more investigation.

Keywords: *eco-friendly demand; Pythagorean fuzzy number; carbon cap-and-trade; preservation technology; partial backlogging; green investment; sensitivity analysis*

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1. Introduction

Consumers are more aware of the contents of the products they purchase. Many consumers are looking for products with herbs versus chemicals, and companies are taking steps to do so by reformulating their products or marketing their credentials or investing in cleaner operations. This change in demand pattern, which conventional models of demand do not account for, is a challenge for inventory managers. As usual, the amount sold will vary with price, and with advertising; but it will also vary with the "mix" of the product. A model that is to be used to guide a new product that is green must therefore include composition as a factor determining demand and must also include considerations of the environmental cost of stockholding and handling the product, since a firm that sells green products is expected to run a green operation.

The model employed in this paper has been developed for exactly this purpose in a doctoral thesis dealing with inventory systems with preservation technology. Its demand function is directly related to a rise in the stock of the product, intensity of advertising, and herbal content and inversely related to its price, chemical content. Preservation can be used to minimize deterioration of the stock. Shortages are permitted and are partially backlogged. Carbon emissions come from the set-up, holding and disposal of degraded units and are governed by a cap-and-trade principle, which involves a fee being imposed on emissions exceeded. The firm can reduce emissions per unit by investing in green technology. Finally, costs of a newly launched product are generally not known precisely, so the cost parameters are represented by triangular Pythagorean fuzzy numbers, where the decision maker can express the "how much" and the "how much not" degree that the cost is acceptable, subject only to the condition that the two degrees are related only by their squares.

The thesis first developed the model, then showed it to be true that the profit function has a unique maximum under a certain condition, and a numerical example was given that was used to give extensive sensitivity tables. Its commentary, however, was generally at the level of direction: a parameter goes up or down profit. If one is going to launch a product, direction is not enough for a manager. The focus is on the size, and particularly which of all the numerous parameters deserves limited management inputs. The published results are therefore treated as data in this paper and four questions are asked. What is the behaviour of the stock of the model and does the optimum reported match it? What is the split between profit gained from sustainability measures and preservation of the resource? What are the margins of manoeuvre for each of the parameters in terms of their influence on profit? What does the counter-intuitive conclusion – particularly for the carbon tax – imply?

The contributions are the following. Closed form stock path is obtained from the model governing

equation and is demonstrated to be the same as the reported order quantities within 0.003, giving an independent check on the numerical results obtained from the model. The profit uplift and the reduction of return on green investments are decomposed. The sensitivity tables are used to calculate elasticities and parameters are ranked. The roles of fuzzy attitude parameter and of the herbal to chemical ratio are discussed. We take the same level of care in discussing the carbon-policy findings, due to their unusual direction. As with the two-warehouse model companion analysis, only values reported in the source and arithmetic used by the reader are used.

2. Related work

There has been a considerable expansion of research on inventory with environmental constraints. Hasan et al. [7] have compared carbon-tax, cap-and-trade, and strict-limit regimes under a situation where firms can invest in technology that reduces their emissions, and concluded that the kind of regulation affects the optimal stock level, as well as the incentive for investment. Singh and Mishra [19] investigated the joint replenishment and substitution policy for non-instantaneously deteriorating items and analyzed carbon emission. Mashud et al. [13] presented an inventory model with sustainable production for imperfect products, product deterioration and controllable emissions, while Sarkar and Chung [18] demonstrated the impact of renewable energy on reducing emissions in a flexible production system. Lu et al. [11] studied the multistage production-inventory system with emission reduction and price-dependent demand in a Stackelberg game. These studies together corroborate that emissions can be factored in the cost structure of an inventory model and that investment can reduce emissions.

In parallel, the demands specifications have been enriched. Advertisement-dependent demand and imperfect production were considered by Manna et al. [12]. Rad, Khoshalhan, and Glock [17] analyzed a two-stage chain under price-sensitive demand but advertisement-sensitive demand, and Cárdenas-Barrón et al. [2] dealt with a nonlinear chain of stock-dependent demand and stock-dependent holding cost and trade credit. Pervin, Roy and Weber [16] studied a 2-level hybrid case with price- and stock-dependent demand, Mishra, Wu and Tseng [14] analysed a hybrid case of price-and stock-dependent demand with remanufactured product, and Chen et al. [3] studied price and replenishment problems involving stock-, time- and price-dependent demand. Das et al. [4] considered the case of a system where the demand depends on the price and partial backlog occurs. The general rule in the lesson was that realistic demand requires multiple levers to be effective. None of the above-mentioned works, however, use the ingredient composition of the product as a lever, which the eco-friendly demand of the present model fills.

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In the context of a time-varying response of deterioration to expenditure, preservation technology has been studied since that of Hsu, Wee, and Teng [9] and was studied by Dye and Hsieh [5] and Hsieh and Dye [8] as a decision variable. Uncertainty has been dealt with in terms of fuzzy sets, introduced by Zadeh [21] and extended by Atanassov [1] to intuitionistic fuzzy sets where membership and non-membership degrees add up to at most one. Yager [20] loosened the requirement to a condition on the sum of squares in order to allow for more flexibility in the representation of pairs, and more flexibility in generating hesitation, which led to the creation of Pythagorean fuzzy sets and operations and decision methods for these sets developed by Zhang and Xu [22]. Pal, Mahapatra and Samanta [15] and Garai, Chakraborty and Roy [6] and others used fuzzy parameters in inventory models and Kumar et al. [10] used fuzziness and sustainability. The current model seems to be one of the first models that introduces Pythagorean fuzzy costs in a sustainable inventory system with eco-friendly demand.

3. The model

3.1 Demand, deterioration and backlogging

The demand for the product at a stock level q, number of ads A, and price p is $D = (a + bq)\cdot A^{\nu}\cdot\gamma\cdot e^{((\alpha-\beta)\rho)}\cdot p^{(-\delta)}$ where a, b, γ and ρ are all positive and $A > 1$ and α, β, ν and δ are between 0 and 1. The parameter α represents the relative amount of herbal ingredients, and the parameter β represents the relative amount of chemical ingredients; the exponential is greater if the product has a higher amount of herbal ingredients than chemical ingredients. The effects of advertising are captured by the parameter ν while the sensitivity of demand to price is captured by the parameter δ. $F = A\cdot\nu\cdot\gamma\cdot e^{((\alpha-\beta)\cdot\rho)}\cdot p^{-\delta}$ is convenient to write as $F = A^{\nu}\cdot\gamma\cdot e^{((\alpha-\beta)\cdot\rho)}\cdot p^{-\delta}$, which separates the part of the demand function that depends on the stock from the part that depends on marketing and product composition. The elasticity of F with respect to price is $-\delta$ and with respect to advertising number is ν – this implies that the sensitivity parameters are read as demand elasticities.

The rate of deterioration is given by the function $\theta = \theta_m\cdot e^{(-a_1\xi)}$ where $0 < \theta_m < 1$ and $a_1 > 0$ and can be controlled with investment expenditures ξ per unit time. Shortages are when stock runs out at time t_i and are backlogged at a fixed rate f. The length of a cycle is T. Let Q_1 be the initial stock, Q_2 be the quantity which remains backlogged at the end of the cycle and order quantity be $Q = Q_1 + Q_2$.

3.2 Carbon emissions and green investment

Each set up, holding of stock, and disposal of deteriorated units includes emissions. The cap-and-trade rule is a rule that imposes a cap of Y on how much a firm can emit per unit time and a tax

of φ on the number of kilograms of emissions in excess of the cap, which is an alternative to the tax on every kilogram of emissions. The firm could invest G units in green technology, which could be in the form of efficient machinery or energy sources. The impact of a zero emission rate at low G is captured by a reduction fraction that is zero if G is zero, rises with G, is concave and tends to a maximum η for high G; this latter maximum efficiency is represented by a parameter m that reflects the efficiency with which technology implements the zero-emission standard. The properties are given but not the formula, an example of a function that has these properties is given as η(1 − e^(−mG)). The cost of the green investment per cycle is GT.

3.3 Pythagorean fuzzy costs and their defuzzification

The triangular Pythagorean fuzzy numbers (TPFNs) are used to represent six cost parameters, holding cost, deterioration cost, lost-sales opportunity cost, purchase cost, shortage cost and ordering cost. Each TPFN is a triple that contains a membership function, which represents how acceptable the value is, and another triple that contains a non-membership function, which represents how unacceptable the value is. For example, the purchase cost is given by the pair of triples (22, 40, 48) and (20, 40, 50), and the holding cost by (10, 15, 18) and (8, 15, 20). The pair is a valid Pythagorean number if at every point the sum of the squares of the two degrees is at most, one.

The source offers a probability based ranking to obtain crisp values. To give high weight to values that are strongly accepted and weakly rejected, two density functions are constructed, one proportional to the membership function, and the other to the complement of the non-membership function. A convex combination with weight λ, where λ is between zero and one, yields a single density and a defuzzified cost of that density is obtained with the aid of a Mellin transform. The parameter λ, in other words, measures the extent to which the information about the membership is trusted by the decision maker over the information about the non-membership. The procedure produces for the base value λ = 0.6, among other things, the values of the holding cost $14.28 and the holding cost per unit $36.40, which are represented in the numerical example.

4. Analytical properties

4.1 Closed-form stock path

When the stock is not being used, it decreases because of the deterioration and demand, and hence dq/dt = −(a + bq)F − θq, q(t_i) = 0. Writing s = bF + θ for the combined proportional depletion rate, the solution is q(t) = (aF/s)·(e^(s(t_i − t)) − 1) for 0 ≤ t ≤ t_i. The initial stock follows by setting t = 0, giving Q_1 = (aF/s)·(e^(s·t_i) − 1). The backlog increases at the rate afF during the shortage interval, and then becomes (T − t_i)·afF

at the end of the interval. These equations are consistent with the equations given in the source for the shortage phase, and give a closed form of the equation for the stocked phase, which is only implicitly given in the source.

The closed form allows an independent check of the reported optimum. With the base parameters (a = 150, b = 0.1, A = 50, ν = 0.4, γ = 2, α = 0.8, β = 0.2, ρ = 1.2, p = 55, δ = 0.6, f = 0.9), the factor F equals 1.7746. With preservation spending ξ = 5 and a_1 = 0.5, the controlled deterioration rate is 0.1·e^(−2.5) = 0.00821, so that s = 0.1855. Inserting the reported t_i = 0.597481 and T = 0.744975 gives Q_1 = 168.197 and Q_2 = 35.335, which match the reported 168.197 and 35.3348 and sum to 203.532. Without preservation the deterioration rate is 0.1, s = 0.2774, and the reported t_i = 0.473401 and T = 0.670989 yield Q_1 = 134.663 and Q_2 = 47.335, again matching the reported values. The acceptance of the third decimal validates the closed form as being faithful to the model and shows that the reported optimum is consistent. Also it shows that the total order quantity of the sustainable case is 203.582 and its members 203.532, and there is a difference of 0.05, which seems to be a transposition error; the sum is used.

4.2 What the closed form shows

The solution to the equation for s is an instructive decomposition. If stored at the base values, then the stock-dependent demand term bF equals 0.1775 and with preservation the rate for deterioration equals 0.0082, which means that preservation causes only 4.4 per cent of the depletion rate, while the rest is due to demand. The preservation rate accounts for 36 per cent of the depletion rate without preservation. This is the reason why, despite the reduction of 92 per cent in the deterioration rate, the stocked interval increases by 26 per cent (from 0.4734 to 0.5975 years), even though the combined rate of deterioration s decreases by 33 per cent (from 0.2774 to 0.1855). It also demonstrates why there is an initial rise of around 25 per cent in the stock, from 134.7 to 168.2 units, when the depletion rate is reduced: a lower rate of depletion requires a larger stock to span a longer stocked interval.

The shortage interval T − t_i reduces from 197.6 to 147.5 years, or by 25.4 per cent, and since Q_2 is proportional to the shortage interval, the backlogged quantity falls from 47.3 to 35.3 units by the same amount. The length of the cycle increases by 11 per cent (from 0.671 to 0.745 years). The profit comparisons in the next section are underpinned by these structural effects caused by the mix of preservation and other sustainability measures. They also demonstrate that preservation does not work on profit directly by saving on degraded products – results below show that the elasticity of deterioration cost is approximately 0.004 – but only via the stocked interval and the shortage interval.

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4.3 Concavity

The source shows that there is a unique global maximum of the average profit function of the two decision variables, the stock-out time t_i and the cycle length T, under a given inequality. The proof is based on a positive affine denominator of the profit function and a strictly concave numerator of the profit function, and uses the theory of pseudo-concave functions. The inequality must be fulfilled by a combination of holding costs, purchase costs and costs due to deterioration and emission, which amounts to approximately 23 at the basic parameters, while the right-hand side is approximately 10. For the purpose of the sensitivity analysis the margin is important because the optimum will not jump in the parameter ranges studied, but will remain unique everywhere, such that the elasticities reported below will reflect the change of a single well-defined optimum and not a jump between local solutions.

5. Value of the sustainability measures

Table 1: Optimal solution with and without green investment and preservation technology

Environment	t_i* (yr)	T* (yr)	Profit TAP*	Q_1*	Q_2*	Q* (Q_1* + Q_2*)
Green and preservation	0.597481	0.744975	3,063.43	168.197	35.335	203.532
Without green and preservation	0.473401	0.670989	2,714.41	134.663	47.336	181.998
Change (per cent)	+26.2	+11.0	+12.9	+24.9	−25.4	+11.8

Table 1 indicates that the sustainable environment brings the optimum average profit from 2,714.41 to 3,063.43 and the difference is 349.02 or 12.9 per cent. The length of the interval of stock increases by 26 per cent, the cycle increases by 11 per cent, the initial stock increases by 25 per cent, while the backlogged amount decreases by 25 per cent. The pattern is that of a firm which, because of slower depletion and lower holding and emission cost, has the ability to hold more stock for longer and thus has a shorter period of shortage in each cycle. The benefit of a shorter shortage period comes in two ways: a fraction of the unmet demand, f = 0.9, will be backlogged and there will be more sales made from stock. This is in line with the source's conclusion that a sustainable environment can contribute to waste and emission control and increase profits.

The table doesn't distinguish between the contributions of preservation and green investment. The source provides the ways to do that in a second table, where the green investment G is changed but the other parameters are kept at their default values, such as preservation. Table 2 is the same as Table 1, with the marginal gain of each extra dollar of G included.

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Table 2: Optimal profit as the green investment G varies

G (US$/unit)	t_i* (yr)	T* (yr)	Profit TAP*	Gain over previous step	Cumulative gain over G = 0
0	0.592295	0.744918	3,032.58	—	—
1	0.595060	0.744936	3,051.32	+18.74	+18.74
2	0.596135	0.744950	3,057.94	+6.62	+25.36
3	0.596708	0.744960	3,060.99	+3.05	+28.41
4	0.597063	0.744967	3,062.49	+1.50	+29.91
5	0.597305	0.744972	3,063.20	+0.71	+30.62
6	0.597481	0.744975	3,063.43	+0.23	+30.85
7	0.597614	0.744978	3,063.37	−0.06	+30.79
8	0.597718	0.744980	3,063.10	−0.27	+30.52

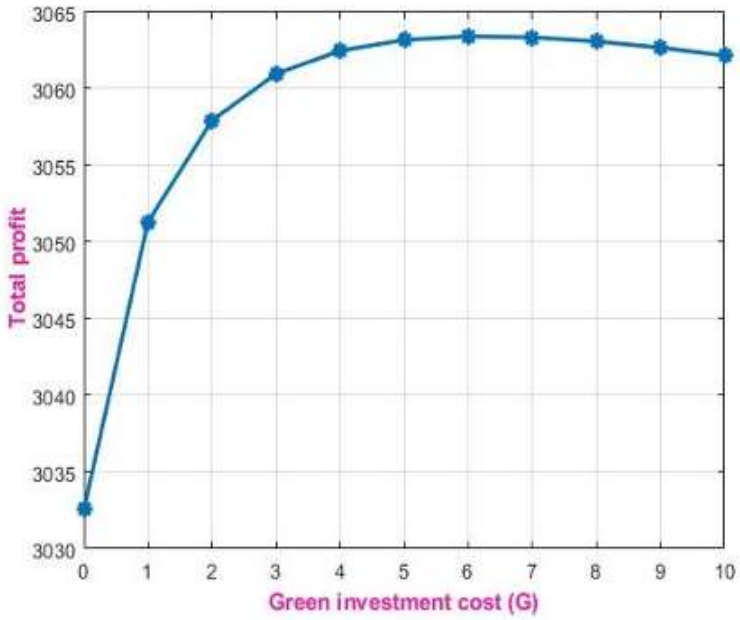


Figure 1: Optimal profit and marginal gain as green investment G increases

Source: *Plotted from Table 2.*

The returns are strong and diminishing as illustrated in Table 2 and Figure 1. The increase in profit is 18.74 when the dollar is green, and the increases are approximately half for every additional dollar in the green dollar investment, until G = 6 is only 0.23 and G = 7 is negative. As theory suggests, the profit function is level around the optimum at G = 6 (where the marginal gain has almost disappeared) and the loss beyond the optimum is not significant. A manager who invests a little under the optimum, say G = 4, realizes 96.9 per cent of the potential gain from investing in green assets, while a manager who invests G = 3 receives 92 per cent. The characteristic of the exponential response to emissions reduction is the

halving pattern, and that suggests that the budget for green investments should be oriented towards the big initial gains, rather than the plateau.

The other interesting result is scale. The overall increase in gain as G increases from 0 to 6 is 30.85; this represents 1.0 per cent of the profit when G = 0. Given the table structure, if the G = 0 row keeps the preservation investment, then the 349.02 uplift in Table 1 is split into 318.17 (91.2 per cent) due to the preservation investment and the other differences between the two environments, and 30.85 (8.8 per cent) due to green investment. The decomposition is shown in figure 2 along with the impact of eco-friendly demand which is discussed later. The decomposition should be taken with the caveat that the source does not explicitly say what the G = 0 row contains. Despite this, it shows that in this model sustainability is primarily a financial matter of preservation, and the green investment can be argued only on the basis of the environmental impact and its role in compliance, to a limited extent a financial impact.

6. Parameter leverage

Each parameter in the sensitivity tables of the source has a variation of 10 and 20 per cent. The results enable us to calculate arc elasticities, which are the difference between the percentage profit change at 10 per cent and at -10 per cent divided by 20. Table 3 gives all the elasticities in rank order by size, and the expected profit changes at twenty per cent increases and decreases are shown in Figure 2.

Table 3: Sensitivity of optimal profit to parameter changes

Parameter	Profit change at −20% (%)	Profit change at +20% (%)	Arc elasticity (±10%)	Rank by |elasticity|
Selling price p	−93.08	+74.81	+4.14	1
Purchase cost C_3	+65.29	−64.58	−3.25	2
Price sensitivity δ	+83.86	−46.63	−3.13	3
Advertising impact ν	−33.36	+48.86	+2.02	4
Demand scale a	−24.04	+24.76	+1.22	5
Holding cost C_1	+6.73	−5.65	−0.31	6
Carbon tax φ	−1.02	+1.03	+0.05	7
Lost-sales cost C_p	+0.90	−0.82	−0.043	8
Shortage cost C_s	+0.82	−0.59	−0.034	9
Emission factor η	−0.24	+0.24	+0.012	10
Deterioration cost C_2	+0.08	−0.08	−0.004	11

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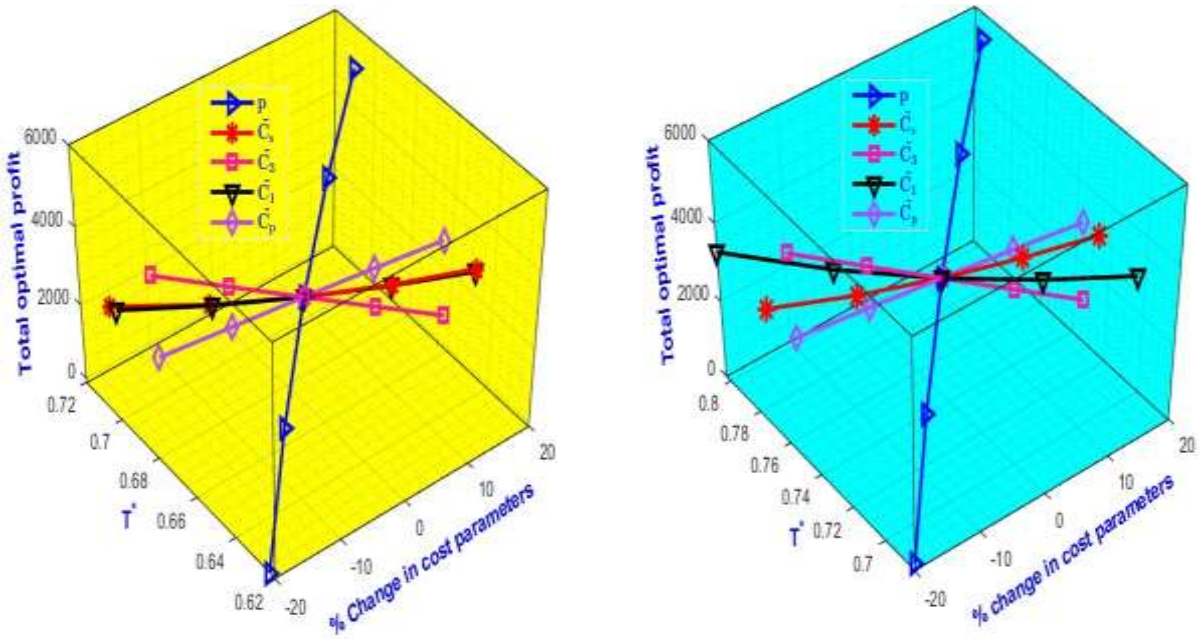


Figure 2: Change in optimal profit for ±20% changes in each parameter

Source: *Plotted from Table 3.*

The five parameters at the top of Table 3 are one to two orders of magnitude larger than the rest, or more than a factor of a million larger. Selling price, purchase cost and price sensitivity have elasticities with magnitudes above three, advertising impact above two and the demand scale above one. In a 20% price decrease the profit increases by 84% and in a 20 % price increment the profit is reduced by 75% when compared with the price decrease, indicating high sensitivity of the profit function with respect to commercial parameters. The gears stem from the thin margin of the units. The price is $55 at the base values and the purchase cost is $36.40, which leaves a margin of $18.60 before holding and other costs. This ratio, 1.5, is of the same order as the ratio of the reported elasticities, 4.14 to 3.25 or 1.27, and the difference can be attributed to the feedback of demand and the cycle length not included in the margin argument.

Table 3 shows the signs that correspond to the economic logic. The low value of the price elasticity of demand, δ, makes the price effect dominate the profit increase, which, for low values of δ, would more than offset the loss in demand. The profit is decreasing when the price is sensitive and its purchase and holding costs are increasing and it is increasing when the impact of advertisement is increasing and the scale of demand is increasing. The asymmetry in the price sensitivity row (profit decreases by 47 per cent if there is a 20 per cent increase in price sensitivity, while it increases by 84 per cent if there is a 20 per cent decrease in price sensitivity) is due to the convexity of the power-law demand function in the price sensitivity δ. The asymmetry in the price row

(negative 93 per cent, positive 75 per cent) reflects the inverse sign of the curve, and suggests that the model is more sensitive to a price decrease than a price increase of the same magnitude.

The parameters listed at the bottom of the table are essentially "passive" parameters. So the holding cost is −0.31, while the magnitudes of the shortage cost, the lost-sales cost, the emission factor and the deterioration cost are all less than 0.05. For deterioration cost, this is similar to the closed form cost analysis of Section 4.2, which shows that the cost for controlled deterioration is approximately four per cent of the depletion cost; for shortage and lost sales cost, it is similar to the short shortage interval of about 0.15 years. The point of interest is that a launch team should work on the price, sourcing and advertising, and not improving its estimates of the minor cost factors. The further value of the inertness is that the parameters are fuzzy numbers, and the treatment of fuzziness for minor parameters has little effect on the decision, whereas for the major ones, such as price and purchase cost in particular, the treatment of fuzziness is where the uncertainty matters.

7. Fuzzy attitude and eco-friendly demand

Table 4: Optimal profit for different values of the defuzzification parameter λ

λ	t_i* (yr)	T* (yr)	Profit TAP*	Gain over previous step	Q* (Q_1^* + Q_2^*)
0	0.590854	0.742191	2,892.01	—	202.483
0.2	0.593049	0.743111	2,949.15	+57.14	202.830
0.4	0.595258	0.744039	3,006.27	+57.12	203.179
0.6	0.597481	0.744975	3,063.43	+57.16	203.532
0.8	0.599717	0.745919	3,120.63	+57.20	203.888
1	0.601967	0.746871	3,177.88	+57.25	204.246

The profit is essentially linear in λ (Table 4) with each increment of 0.2 corresponding to an increment of between 57.12 and 57.25, and a range of 285.87 or 9.9 per cent between the extremes. The linearity as implied by the construction is due to the fact that the defuzzified cost is a convex combination between two expected costs (with a weight λ), and the profit is almost linear in the costs over the small range of interest. The outcome is comforting and it is also a warning. It's comforting because the optimum decisions variables (t_i* and T*) change by less than 2 per cent and 1 per cent respectively with the choice of λ. It is a caution because the reported profit will differ somewhat with the subjective weight so reported profits obtained using different values of λ should not be compared. If the decision maker wants to be conservative, he should use the low

end of the range, since a larger λ leads to higher profit here; the source's suggestion to use λ depending on the market conditions is best understood as a suggestion that the model be tested at the extremes.

Table 5: Optimal solution under different herbal-to-chemical ingredient scenarios ω

ω	t_i* (yr)	T* (yr)	Profit TAP*	Q_1*	Q_2*	Q*
0.1	0.929017	1.31569	652.62	80.526	29.273	109.798
0.5	0.857248	1.19576	858.82	90.286	31.051	121.337
1	0.777642	1.06138	1,185.44	104.554	33.086	137.640
2	0.708321	0.94251	1,637.00	121.683	72.257*	193.941*
4	0.677252	0.88837	1,918.93	131.567	35.287	166.854

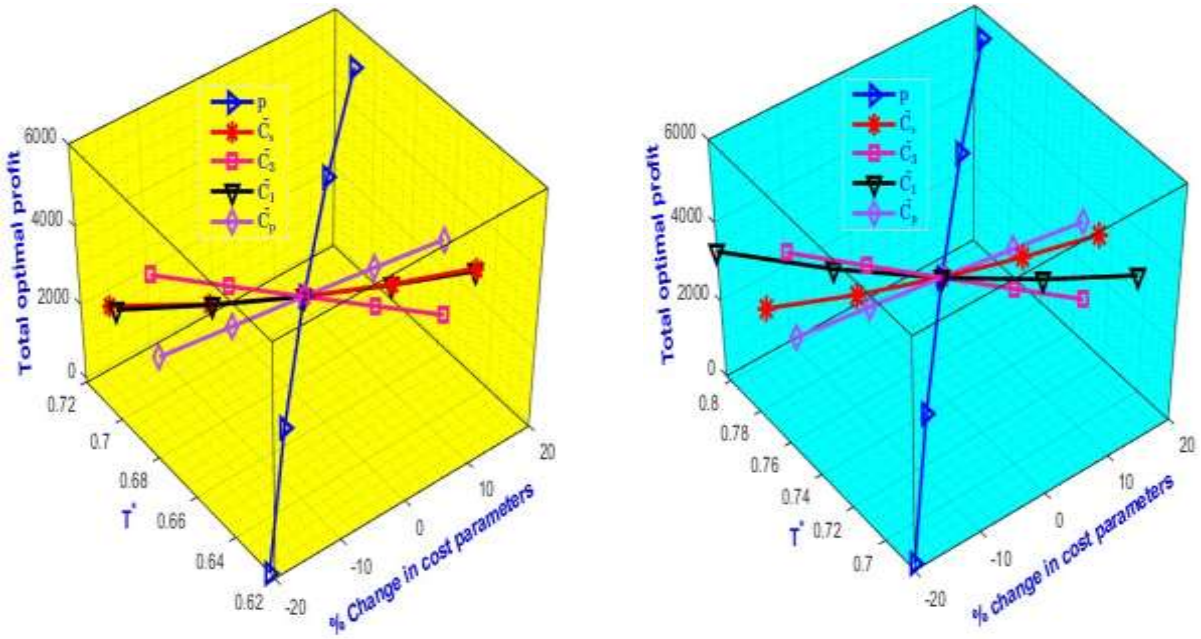


Figure 3: Decomposition of the profit uplift (left) and optimal profit under herbal-to-chemical scenarios (right)

Source: *Plotted from Tables 1 and 2 (left) and Table 5 (right).*

Table 5 shows the effects of the ratio of herbal to chemical content, here denoted through the parameter ω from the source. If ω is less than 1, the product is mainly chemical, and demand is low, profit is low; If ω is equal to 1, the exponential component of F is equal to 1 and the composition is neutral; If ω is greater than 1, the amount of herba increases the demand. The stocked interval decreases from 0.93 to 0.68 years as sales speed up, while profits increase monotonically from 652.62 to 1,918.93, or 2.94 times.As the sales speed up, the stocked interval is reduced from 0.93 years to 0.68 years, while the profits increase monotonically from 652.62 to 1,918.93, or 2.94 times. The source describes these as three cases, and the table fills in the continuum. The

commercial message is that in the present context, for a given price and advertising level, reformulating the product towards the herbal ingredients is one of the most effective levers at the disposal of the company; and that increasing the proportion of the ingredients from ω = 1 to ω = 2 is able to increase the profit by 38 per cent per unit of product, for the same price and advertising level.

There is a caution in one row of Table 5. The quantity Q_2 corresponding to ω = 2 is printed as 72.257, nearly twice the values of the neighbouring values of ω, between 29 and 35. The row is internally consistent ($Q_1 + Q_2$ = the printed Q), but the short quantity that the model implies (afF(T − t_i)) requires the demand factor F to be approximately 2.3 at ω = 2, while the other rows suggest a variation smoothly increasing from 0.56 to 1.24. Profit and the cycle times are monotone and do not change because of the anomaly but the quantities for this row should be checked again before using. The printed values and marking are left as they are.

8. Carbon policy: a counter-intuitive result

The carbon treatment deserves some special mention as the direction of the carbon tax effect is unexpected. As can be seen in Table 3, a 20 per cent increase in the carbon tax, φ, leads to an increase of +1.03 per cent in profit, while a 20 per cent decrease in the carbon tax leads to a decrease of −1.02 per cent in profit. The source concludes that a high carbon price is good for the environment and the economy. The environmental part of the statement is feasible because a tax provides an incentive to reduce emissions. The economic half is unusual in that a tax on emissions is a cost: in the standard form a higher tax reduces the profit of the firm that emits.

There are two interpretations. The first is that, in the example, the firm's emissions are less than the cap Y = 800 kg per unit time, making the allowance term φYT an entitlement that its profits from a higher price by selling its unused allowance. This is consistent with both the cap-and-trade design and the fact that the impact is actually quite small, but can't be verified with the data provided in the source, which does not state the firm's actual emissions. The second interpretation is that the sign comes about because of the way the allowance cost is included in the profit function. The impact is small in either case, having an elasticity of 0.05, and even smaller for the emission factor η, with an elasticity of 0.012. We therefore recommend that readers should not assume the positive carbon tax effect for the case of a net emitter firm exceeding the carbon cap. A check of the emissions against the cap, and the carbon cost as an additional profit, would clear the issue up.

The emission-related parameters for the graphical analysis of the source act as expected: higher emission during set-up, holding or decay makes the cycle longer or shorter, and takes a little profit as a small percentage of the total. The more salient

aspect is that carbon parameters have comparatively little leverage, relative to commercial parameters, and in this example, the profit of a firm is driven primarily by the commercial parameters — price, sourcing cost, and marketing. The leverage would increase if the cap was tighter, or if the tax were greater, and the model can be used to investigate this. The result shows that, the main reason for a company of this type to practice green practices is not carbon cost, but customer response – via eco-friendly demand.

9. Managerial implications

A company that is introducing an environmentally friendly product can learn a few things. The first is to establish the price and sourcing first. Elasticities of price, purchase cost and price sensitivity are greater in absolute value than three and, among them, decide on whether the product is profitable or not. Reducing the negotiating price by ten per cent increases profit by approximately one-third and is larger than anything that can be gained by refinement of the minor costs.

The second lesson is on advertising and composition. The elasticity of the advertising effect is 2, indicating a strong effect from advertising, and the profit increase for the composition effect was almost 3 times higher in the range studied, indicating the effect of the composition of the production process is also strong. The marketing effort, spent on establishing the herbal credentials of the product and then advertising them, is likely to be most beneficial for a new product, because both the herbal share and the number of ads are multiplicative factors in the demand function. Note that the advertising number has an advertisement cost, and that the net value is dependent on the cost of the advertisement (a small portion of the total cost, as is the case in the example).

The third lesson relates to expenditure on technology. Preservation gets a bigger cut of the benefits of sustainability efforts (around 90 percent in the decomposition above) and should be the first to be added. The initial investment return is high but rapidly tapers off; investment should focus on first few dollars per unit; the shape of the profit curve around the optimum is shallow so the exact optimum doesn't need to be found. The fourth lesson is that the carbon tax should not be expected to be a cash cow (in the above sense) and the firm should establish its stance towards its carbon cap in advance before deciding its conclusions. The 5th is that the attitude for the defuzzification is to be set and reported since the profit in between these values differs by around 10 per cent.

10. Limitations

The results are based on one numerical example only, and are based on one-at-a-time perturbations, without considering interactions (e.g. the effect of the price on the profit is likely to depend on the number of ads and on the share of herbs). The profit uplift is decomposed depending on an interpretation of the structure of the source table.

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The elasticities are arc estimates of ten per cent change in price and change in price sensitivity, and are only accurate within the ten per cent range; the difference in the price and price sensitivity response to twenty per cent changes, both positive and negative, indicates that the response is nonlinear. In most of the analysis, the number of ads is assumed to be fixed but it is an obviously decision variable, and the amount of green investment per unit and the amount of preservation spending is also assumed to be fixed. The formula for emission reduction is given by properties, and not by an explicit formula; hence, the statements made about the formula are limited to the properties. Finally, some printed values, in particular the values in the row corresponding to ω = 2 and the order quantity in the base case, exhibit irregularities that we have highlighted, but which do not affect the conclusions that are drawn.

Future work might consider advertising, price, and preservation as decision variables, as well as green spending; in this case, the elasticities calculated here would suggest where the gradient information is most useful. If the cost is fuzzy, then a stochastic approach to demand would be used to check if the fuzzy cost is sufficient. The model could be applied to an actual product launch where the proportions of herbal and chemical content are known, and emissions are recorded, to test the demand specification and answer the question of the carbon-tax sign.

11. Conclusion

An inventory model for a new product introduced in the market which is eco-friendly and whose demand is a function of stock, price, advertising and ingredient composition, preservation controlled deterioration, partial backlogging, a carbon cap and trade rule, green investment and Pythagorean fuzzy cost, has been analysed in this paper. A closed-form stock path derived here reproduces the reported optimum to three decimals and helps to understand the reason for preservation increasing the stocked interval and decreasing the shortage interval. The sustainability measures boost optimal profit by 12.9 per cent, with about 91 per cent of the benefit coming from preservation and about 9 per cent from green investment whose returns decline by approximately half when the investment is increased by US$1 per unit, and reach their highest point of US$6 per unit. Profit is influenced by selling price, purchase cost, price sensitivity, advertising impact and scale of demand with elasticities ranging from 1.2 to 4.1 in magnitude, and is almost insensitive to carbon tax, emission factors and deterioration cost. The defuzzification attitude changes by almost ten per cent over its range and a higher herbal-to-chemical ratio increases the profit by nearly three times. The positive carbon-tax effect indicator is small, dependent and we suggest it be confirmed before applying it. Combined, the results provide a ranked list for a launch team to prioritise their attention and offer a specific set of questions to pursue as

part of the next phase of the research for joint optimisation, interaction effects and carbon-policy mechanics.

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